Neutron Matter and Maximum Mass Limits to Neutron Star Radii

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Berkeley, 27 February 2015
Topical Collaboration Meeting
Outline

- Basics of Neutron Star Structure
- Mass Measurements
- Neutron Matter Calculations
- Quartic Contributions to the Nuclear Symmetry Energy
- The Maximally Compact EOS and Mass-Radii Limits
- Neutron Matter Extrapolations and Radii Limits
Neutron Star Structure

Newtonian Gravity:

\[ \frac{dp}{dr} = -\frac{Gm\rho}{r^2}; \quad \frac{dm}{dr} = 4\pi \rho r^2; \quad \rho c^2 = \varepsilon \]

Why $1 - 2\rho_s$? Why $1/4$?

Newtonian Polytrope:

\[ p = K\rho^\gamma; \quad M \propto K^{1/(2-\gamma)} R^{(4-3\gamma)/(2-\gamma)} \]

\[ \rho < \rho_s: \quad \gamma \simeq \frac{4}{3}; \quad \rho > \rho_s: \quad \gamma \simeq 2 \]

Graph showing the relationship between $\rho/\rho_s = 1.0$, $p(\varepsilon)$, $\log_10 p$ (MeV/fm$^3$), and $\rho/\rho_s$ vs. $\varepsilon/\varepsilon_s$.

Maximum mass

\[ p = K\rho^2 \]

\[ R \propto K^{1/2} M^0 \]

\[ p = K\rho^{4/3} \]

\[ M \propto K^{3/2} R^0 \]
Although simple average mass of w.d. companions is 0.23 M\(_\odot\) larger, weighted average is 0.04 M\(_\odot\) smaller.
The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff" (Koranda, Stergioulas & Friedman 1997).

$\epsilon_0$ is the only EOS parameter

The TOV solutions scale with $\epsilon_0$

$w = \epsilon / \epsilon_0$

$y = p / \epsilon_0$

$x = r \sqrt{G \epsilon_0 / c^2}$

$z = m \sqrt{G^3 \epsilon_0 / c^2}$
Extremal Properties of Neutron Stars

The maximum mass configuration is achieved when $x_R = 0.2404$, $w_c = 3.034$, $y_c = 2.034$, $z_R = 0.08513$.

A useful reference density is the nuclear saturation density (interior density of normal nuclei):
$\rho_s = 2.7 \times 10^{14}$ g cm$^{-3}$, $n_s = 0.16$ baryons fm$^{-3}$, $\varepsilon_s = 150$ MeV fm$^{-3}$

- $M_{\text{max}} = 4.1 \left(\frac{\varepsilon_s}{\varepsilon_0}\right)^{1/2} M_{\odot}$ (Rhoades & Ruffini 1974)
- $M_{B,\text{max}} = 5.41 \left(\frac{m_B c^2}{\mu_o}\right) \left(\frac{\varepsilon_s}{\varepsilon_0}\right)^{1/2} M_{\odot}$
- $R_{\text{min}} = 2.82 \frac{GM}{c^2} = 4.3 \left(\frac{M}{M_{\odot}}\right) \text{ km}$
- $\mu_{b,\text{max}} = 2.09$ GeV
- $\varepsilon_{c,\text{max}} = 3.034 \varepsilon_0 \simeq 51 \left(\frac{M_{\odot}}{M_{\text{largest}}}\right)^2 \varepsilon_s$
- $p_{c,\text{max}} = 2.034 \varepsilon_0 \simeq 34 \left(\frac{M_{\odot}}{M_{\text{largest}}}\right)^2 \varepsilon_s$
- $n_{B,\text{max}} \simeq 38 \left(\frac{M_{\odot}}{M_{\text{largest}}}\right)^2 n_s$
- $B_{\text{E, max}} = 0.34 \ M$
- $P_{\text{min}} = 0.74 \left(\frac{M_{\odot}}{M_{\text{sph}}}\right)^{1/2} (R_{\text{sph}}/10 \text{ km})^{3/2} \text{ ms} = 0.20 \left(\frac{M_{\text{sph, max}}}{M_{\odot}}\right) \text{ ms}$
Maximum Energy Density in Neutron Stars

\[ p = s(\varepsilon - \varepsilon_0) \]

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Causality + GR Limits and the Maximum Mass

A lower limit to the maximum mass sets a lower limit to the radius for a given mass.

Similarly, a precise \((M, R)\) measurement sets an upper limit to the maximum mass.

1.4\(M_{\odot}\) stars must have \(R_{1.4} > 8.15M_{\odot}\) using the current observed minimum maximum mass of 2\(M_{\odot}\).

1.4\(M_{\odot}\) strange quark matter stars (and likely hybrid quark/hadron stars) must have \(R > 11\) km.
Maximum Mass and $1.4 M_\odot$ Radii

![Graph showing the relationship between maximum mass ($M_{\text{max}}$) and radius ($R_{1.4}$). The graph includes two curves: one for $s = 1$ and another for $s = 1/3$.](image-url)
What About Realistic EOSs?

It has been proposed that the effective sound speed limit is $c/\sqrt{3}$ (Bedaque & Steiner 2015), in which case $1.4M_\odot$ stars must have $R_{1.4} > 11$ km.

Hybrid quark/hadron stars are realistically at least 1-2 km larger (Alford et al. 2015).

What additional constraints are imposed by our knowledge of the low-density equation of state?
The study of Hebeler & Schwenk (2010) suggested moderate values $40 \text{ MeV} < L < 60 \text{ MeV}$, consistent with but at the lower boundary of the range favored by nuclear experiments.

These results were in substantial agreement with the quantum Monte Carlo neutron matter calculations of Gandolfi, Carlson & Reddy (2012). The chiral Lagrangian calculations have been refined and extended to matter with protons, with proton fractions up to and including symmetric matter are considered.

The symmetry energy coefficients are found to be correlated with the saturation properties for a given parameter set.

There is a small quartic contribution to the symmetry energy.
Neutron Matter Comparisons

GCR

DS

$L$ (MeV)

$S_{v!}$ (MeV)

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Neutron-Rich Matter – Polynomial Fit

\[ x = 0.00 \quad k = 2 \text{--} 5 \]

- model 0: \( KE = 2.99 \times 10^1 \)
- model 1: \( KE = 3.00 \times 10^1 \)
- model 2: \( KE = 2.08 \times 10^1 \)
- model 3: \( KE = 3.01 \times 10^1 \)
- model 4: \( KE = 3.02 \times 10^1 \)
- model 5: \( KE = 3.10 \times 10^1 \)
- model 6: \( KE = 3.19 \times 10^1 \)

\[ \Delta E \text{ (MeV)} \]

- • quartic
- + quadratic

\[ n \text{ (fm}^{-3}\text{)} \]

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Neutron-Rich Matter – Polynomial Fit

![Plot of ΔE vs. n for different models](image)

\[ \Delta E (\text{MeV}) \]

\[ n (\text{fm}^{-3}) \]

- model 0
- model 1
- model 2
- model 3
- model 4
- model 5
- model 6

- quartic
- quadratic
Neutron-Rich Matter – Polynomial Fit

\[ \Delta E \ (\text{MeV}) \]

\[ n \ (\text{fm}^{-3}) \]

<table>
<thead>
<tr>
<th>Model</th>
<th>( S_v )</th>
<th>( S_2 )</th>
<th>( L )</th>
<th>( L_2 )</th>
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<td>6</td>
<td>28.97</td>
<td>28.12</td>
<td>49.36</td>
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</table>

- quartic
- quadratic
Neutron-Rich Matter – Polynomial Fit

\[ x = 0.20 \quad k = 2 \text{--} 5 \]

- Model 0
- Model 1
- Model 2
- Model 3
- Model 4
- Model 5
- Model 6

\( \Delta E \) (MeV)

\( n \) (fm\(^{-3}\))

- Quartic
- Quadratic

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Neutron Matter and Maximum Mass Limits to Neutron Star Radii
Neutron-Rich Matter – Polynomial Fit

\[ \Delta E (\text{MeV}) \]

\[ n (\text{fm}^{-3}) \]

- model 0
- model 1
- model 2
- model 3
- model 4
- model 5
- model 6

\[ x = 0.30 \quad k=2-5 \]

- quartic
- quadratic
Neutron-Rich Matter – Polynomial Fit

\[ x = 0.50 \quad k = 2 - 5 \]

- **model 0** KE = $-8.88 \times 10^0$
- **model 1** KE = $-4.91 \times 10^0$
- **model 2** KE = $-7.18 \times 10^0$
- **model 3** KE = $-1.52 \times 10^0$
- **model 4** KE = $5.34 \times 10^0$
- **model 5** KE = $-1.02 \times 10^1$
- **model 6** KE = $-9.61 \times 10^0$

\[ \Delta E \text{ (MeV)} \]

- quartic
  - \( n_s \) = 0.188
  - \( E_s \) = $-16.87$
  - \( K_s \) = 231.42
- + quadratic
  - \( n_s \) = 0.175
  - \( E_s \) = $-15.56$
  - \( K_s \) = 208.39
  - \( n_s \) = 0.175
  - \( E_s \) = $-15.20$
  - \( K_s \) = 206.16
  - \( n_s \) = 0.166
  - \( E_s \) = $-14.74$
  - \( K_s \) = 195.96
  - \( n_s \) = 0.152
  - \( E_s \) = $-13.62$
  - \( K_s \) = 186.33
  - \( n_s \) = 0.189
  - \( E_s \) = $-16.31$
  - \( K_s \) = 225.65
  - \( n_s \) = 0.137
  - \( E_s \) = $-13.15$
  - \( K_s \) = 193.42
Gandolfi, Carlson & Reddy fit their QMC neutron matter equations of state to the 4-parameter fit:

\[ E_n(u) = au^\alpha + bu^\beta \]

with \( u = n/n_s \) and \( n_s = 0.16 \text{ fm}^{-3} \).

This can also be done with the chiral Lagrangian neutron matter equations of state computed by Drischler, Hebeler & Schwenk.
Neutron Matter Extrapolations and $M - R$
Neutron Matter Extrapolations and $M_{\text{max}} - R_{1.4}$
Neutron Matter Extrapolations and $R_{1.4} - L$